Mental Arithmetic Questions

1. What number is five cubed?
   \[5 \times 5 \times 5 = 125\]

2. A circle has radius \( r \).
   What is the formula for the area of the circle?
   
   \[\text{Area} = \pi r \times r = \pi r^2\]

3. Jenny and Mark share some money in the ratio two to three. Jenny’s share is one hundred and ten pounds. How much is Mark’s share?
   \[\times 55 \quad \frac{2}{3} \quad \times 55 \]
   \[\£165\]

4. The net of a triangular prism is made from triangles and rectangles. How many of each shape are needed?
   
   \[\text{3 rectangles} \quad \text{2 triangles}\]

5. Multiply minus six by minus two.
   \[-6 \times -2 = 12\]
Percentage Change

(a) One calculation below gives the answer to the question What is 70 increased by 9%.
Tick (√) the correct one.

- What is 70 increased by 10%
- What is 70 decreased by 91%
- Find 90% of 70
- 190% of 70
- What is 9% of 70

Choose one of the other calculations.
Write a question about percentages that this calculation represents.

Calculation chosen: 70 x 0.9
Question it represents: What is 9% of 70

Calculation chosen: 70 x 1.9
Question it represents: 70 increased by 9%

Calculation chosen: 70 x 0.09
Question it represents: 70 decreased by 9%

Calculation chosen: 70 x 1.09
Question it represents: 70 increased by 10%

Coffee

A cup of coffee costs £1.75

The diagram shows how much money different people get when you buy a cup of coffee.

Cup of Coffee Costs £1.75

(a) Complete the table to show what percentage of the cost of a cup of coffee goes to the retailers, growers and others.

Show your working

£1.75 = 175p

<table>
<thead>
<tr>
<th></th>
<th>Retailers</th>
<th>Growers</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>44 x 100</td>
<td>25.1%</td>
<td>2.9%</td>
<td>72%</td>
</tr>
<tr>
<td>5 x 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>126 x 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>175</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Must add to 100%
Mental Arithmetic Questions

1. What is one third of three-quarters of one hundred?
   \[ \frac{1}{3} \times \left( \frac{3}{4} \times 100 \right) = \frac{1}{3} \times 75 = 25 \]

2. I’m thinking of a number. I call it \( n \).
   I square my number and then add 4.
   Write an expression to show the result.
   \[ n^2 + 4 \]

3. Twenty-one out of thirty-six pupils said they watched Top of the Pops.
   What angle on a pie chart would show this?
   
   \[ 1 \text{ pupil} = \frac{360^\circ}{36} = 10^\circ \quad \therefore \quad 21 \text{ pupils} = 21 \times 10 = 210^\circ \]

4. There are seven red and three blue balls in a bag. I am going to take a ball out of the bag at random. What is the probability that the ball will be blue?
   \[
   \text{No of Blue} = 3 \\
   \text{Total} = 10
   \]

5. Write a multiple of three that is bigger than one hundred.
   102, 105, 108, ....... or other
Powers

(a) Write the values of k and m.

\[ \begin{align*}
4 \times 4 \times 4 \\
2 \times 2 \times 2 \times 2 \times 2 \times 2
\end{align*} \]

\[ 64 = 8^2 = 4^k = 2^m \]

\[ 8 \times 8 \]
\[ = 4 \times 2 \times 4 \times 2 = \]
\[ = 4 \times 4 \times 4 \times 4 = 4^3 \]

\[ k = \ldots \ldots 3 \ldots \ldots \]
\[ m = \ldots \ldots 6 \ldots \ldots \]

2 marks

(b) Complete the following:

\[ 2^{15} = 32 \ 768 \]
\[ 2^{15} = 2^{14} \times 2^1 \] so \[ 32 \ 768 \div 2 \]

\[ 2^{14} = \ldots \ldots 16384 \ldots \ldots \]

1 mark

Squares

Some numbers are smaller than their squares.

For example: \[ 7 < 7^2 \]

Which numbers are equal to their squares?

\[-1 \times -1 = 1\]
\[0 \times 0 = 0\]
\[1 \times 1 = 1\]

So only 0 & 1

2 marks
Mental Arithmetic Questions

1. I am thinking of a number. I call it \( n \).
I double my number then I subtract three.
Write an expression to show the result.
\[
n \text{ double it} = 2n
\]
Subtract 3
\[
2n - 3
\]

2. What percentage of fifty pounds is thirty-five pounds?
\[
\frac{35 \times 100}{50} = 70\%
\]

3. On average, the driest place on earth gets only nought point five millimetres of rain every year.
In total, how much rain would it expect to get in twenty years?
\[
0.5 \times 20 = 10\text{mm}
\]

4. To the nearest whole number, what is the square root of eighty-three point nine?
\[
83.9 = 81 \quad \therefore \sqrt{81} = 9
\]

5. It takes me one and a half minutes to swim one length of the pool.
How many lengths can I swim in fifteen minutes?
\[
\frac{15}{1.5}
\]
\[
1.5 \times 10 = 15 \quad \therefore 10 \text{ lengths}
\]
Pentagonal Pyramid

The diagram shows the net for a right-pyramid with a regular pentagon as its base.

The net is constructed using five straight lines.

(a) Without measuring, explain why angle a must be 108°

Is a regular pentagon
Interior angles = 3 triangles = 3 x 180 = 540°
Ach angle = \( \frac{540}{5} = 108° \)

(b) Calculate the size of angle b.
You must show your working

Isosceles

\[ 180 - 108 = 72 \]
\[ 72 \div 2 = 36 \]

\[ \ldots..36°\ldots\ldots. \]

1 mark

(c) On these nets, the point marked P lies on the perpendicular bisector of a side of the pentagon.

On side CD of the regular pentagon below, use compasses and a straight edge to construct the perpendicular bisector.

You must leave in your construction lines.

2 marks
Mental Arithmetic Questions

1. Tariq won one hundred pounds in a maths competition. He gave two-fifths of his prize money to charity. How much of his prize money, in pounds, did he have left?

   \[
   1 - \frac{2}{5} = \frac{3}{5}
   \]

   \[
   \frac{3}{5} \text{ of } £100 = \frac{3}{5} \times £100 = £60
   \]

2. What is three point nine divided by two?

   \[
   3.9 \div 2 = 3 \div 2 = 1.5
   \]

   \[
   0.9 \div 2 = 0.45
   \]

   \[
   1.95
   \]

3. The instructions for a fruit drink say to mix one part blackcurrant juice with four-part water. I want to make one litre of this fruit drink. How much blackcurrant juice should I use? Give your answer in millimetres.

   \[
   1 \text{ part} \therefore 4 \text{ parts } 1000\text{ml} = 1 \text{ litre}
   \]

   \[
   5 \text{ parts in } 1000\text{ml} \therefore \text{ each } \frac{1000}{5} = 200\text{ml}
   \]

   \[
   \therefore 200\text{ml} \text{ of blackcurrant per part}
   \]

4. What is half of two-thirds?

   \[
   \frac{1}{2} \text{ of } \frac{2}{3} = \frac{1}{3} + \frac{1}{3}
   \]

5. The population of the United Kingdom is about fifty-nine million. Write this number in figures.

   \[
   59\ 000\ 000
   \]
Running Machine

Kali uses a running machine to keep fit.

The simplified distance-time graph shows how she used the machine during one run.

(a) Between 0930 and 0940, what was her speed in kilometres per hour?

\[
\begin{array}{c|c}
\text{Km} & \text{Mins} \\
1 & 10 \times 6 \\
& 60 \\
\hline
& 6\text{km/h}
\end{array}
\]

1 mark

(b) Throughout the run, for how many minutes did she travel at this speed?

9.30 - 9.40 and 9.50 - 10.00
10mins + 10mins

20mins

1 mark

(c) At 0940, she increased her speed.
By how many kilometres per hour did she increase her speed?

09.40 - 9.50 travelled 2\(\frac{1}{2}\) km

\[
\begin{array}{c|c}
\text{Mins} & \text{Km} \\
10 & 0.5 \\
60 & 9 \\
\hline
& 6
\end{array}
\]

9 is 3km/h more than 6

3km/h

1 mark

(d) On another day, Kali started running at 0935. She ran for 24 minutes at a constant speed of 10 kilometres per hour.

\[
\begin{array}{c|c}
10\text{km} & 60\text{mins} \\
2.5 & 15\text{mins} \\
\hline
& 4
\end{array}
\]

2.5km in 15mins

Show this information on the graph

2 marks

Use the graph to answer these questions.
Mental Arithmetic Questions

1. What is three-fifths of forty pounds?
   \[ \frac{3}{5} \text{ of } £40 = £8 \text{ so } \frac{3}{5} \times 8 = £24 \]

2. The longest bone in the human body is in the leg. The average length of this bone in a man is fifty centimetres. In a woman it is ten percent less. What is the average length of this bone in a woman?
   - 50cm
   - 10% less than 50cm
   - 10% of 50 = 5
   - \[ 50 - 5 = 45 \text{cm} \]
   - or woman is 100% - 10% = 90%
   - 90% of 50 = 45cm

3. Using three as an approximation for \(\pi\), what is the area of a circle with radius five centimetres?
   \[ a = \pi \times r^2 = 3 \times 5^2 = 3 \times 25 = 75 \text{cm}^2 \]

4. I am thinking of a two-digit number that is a multiple of eight. The digits add up to six. What number am I thinking of?
   - Multiples of 8. 16, 24, 32...
   - \( 2 + 4 = 6 \) \( \Rightarrow \) 24

5. I am thinking of a number. I call it \( n \). I add five to my number.
   Write an expression to show the result.
   \[ N + 5 \]
Loci

The diagram below shows two points A and B that are 6cm apart.

Around each point are six circles of radius 1cm, 2cm, 3cm, 4cm, 5cm and 6cm. Each circle has either A or B as its centre.

(a) On the diagram, mark with a cross any points that are 4cm away from A and 4cm away from B.
This is where 4km circles cross

1 mark

(b) Now draw the locus of all points that are the same distance from A as they are from B
Join the 2 points
Make sure they extend beyond the circles

1 mark

Rearrange

The subject of the equation below is p

\[ P = 2(e + f) \]

Rearrange the equation to make e the subject

Method 1

\[ P = 2(e + f) \quad \text{or} \quad P = 2(e + f) \]
\[ P = 2e + 2f \]
\[ P - 2f = 2e \]
\[ p - 2f = e \]
\[ \frac{p - f}{2} = e \]

2 marks
Mental Arithmetic Questions

1. Five percent of a number is 8. What is the number?

\[ \times 10 \]
\[ 5\% = 8 \]
\[ \times 2 \]
\[ 50\% = 80 \]
\[ \times 100\% = 160 \]

2. A fair spinner has eight equal sections with a number on each section. Five of them are even numbers. Three are odd numbers. What is the probability that I spin an even number?

\[ 5 \text{ - no of even} \]
\[ 8 \text{ - Total no.} \]

3. I can make a three-digit number from the digits two, three and four in six different ways. How many of these three-digit numbers are even?

2, 3, 4

\[
\begin{array}{ccc}
234 & 324 & 432 \\
243 & 342 & 423 \\
\end{array}
\]

4 even

4. What is the volume of a cuboid measuring five centimetres by six centimetres by seven centimetres?

\[ L \times W \times H = 5 \times 6 \times 7 \]
\[ 30 \times 7 = 210 \]

5. What is the remainder when you divide three hundred by twenty-nine?

\[ 29 \times 10 = 290 \]
\[ \therefore 10 \text{ is remainder} \]
Evens or Odds

(a) \( m \) is an odd number.

Which of the numbers below must be even, and which must be odd?

Write 'odd' or 'even' under each one.

<table>
<thead>
<tr>
<th>( 2m )</th>
<th>( m^2 )</th>
<th>( 3m - 1 )</th>
<th>( (m - 1)(m + 1) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td>Odd</td>
<td>Even</td>
<td>Even</td>
</tr>
</tbody>
</table>

2 marks

(b) \( m \) is an odd number.

Is the number \( \frac{m + 1}{2} \) odd, or even, or is it not possible to tell?

Tick (\( \checkmark \)) the correct box.

Odd | Even | Not possible to tell

Explain your answer.

\[ \text{If } m \text{ is odd } m + 1 \text{ is even} \]

\[ \frac{m + 1}{2} \text{ to be even, } m + 1 \text{ must be a multiple of } 4. \]

Factors Again

(a) Ring the expression below that is the same as \( y^2 + 8y + 12 \)

Need factors of +12 which add to +8

\[ \begin{align*}
\text{Factors} \\
1 & \quad 12 \\
-1 & \quad -12 \\
2 & \quad 6 \\
-2 & \quad -6 \\
4 & \quad 3 \\
-4 & \quad -3
\end{align*} \]

\[ \begin{align*}
(y + 3)(y + 4) & \quad (y + 7)(y + 1) \\
(y + 1)(y + 12) & \quad (y + 3)(y + 5)
\end{align*} \]

\( \checkmark (y + 2)(y + 1) \)

1 mark

(b) Multiply out the expression \((y + 9)(y + \_\)\)

Write your answer as simply as possible

Partition each bracket

\[
\begin{array}{|c|c|c|}
\hline
y & 9 \\
\hline
y & y^2 & + 9y \\
+2 & +2y & +18 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
y^2 & +9y & 2y \\
\hline
& & +18 \\
\hline
\end{array}
\]

\[ y^2 + 11y + 18 \]

2 marks
Mental Arithmetic Questions

1. Twenty-five percent of a number is seven. What is the number?

   \[ x^2 \quad 25\% \text{ is } 7 \quad x^2 \]
   \[ x^2 \quad 50\% \text{ is } 14 \quad x^2 \]
   \[ x^2 \quad 100\% \text{ is } 28 \quad x^2 \]

2. There are fourteen girls and thirteen boys in a class. What is the probability that a pupil chosen at random will be a girl?

   Total = 14 + 13 = 27
   Girls = 14
   Total 27

3. The first even number is two. What is the hundredth even number?

   \[ 2 \times 100 = 200 \]

4. The mean of two numbers is 8. One of the numbers is two. What is the other number?

   Total must be \(8 \times 2 = 16\)
   \[ ? + 2 = 16 \quad : ? = 14 \]

5. How many edges are there on a square based pyramid?

   4 on base
   4 on sides
   = 8
**Angles**

This pattern has rotation symmetry of order 6

(a) What is the size of angle $w$?

Show your working.

There are 6 angles

26°

so $6 \times 26 = 156°$

Angles at a point = 360°

So $360 - 156 = 204°$

204° to share between 6 angles

$204 \div 6 = 34°$

(b) Each quadrilateral in the pattern is made from two congruent isosceles triangles.

What is the size of the angle $y$? Show your working.

This triangle

$154°$

$154°$

$52°$

$360° - 154° - 154° = 360° - 308°$

$= 52°$

**Television**

A headteacher wants to choose a pupil from year 7, 8 or 9 to appear on television.

The headteacher gives each pupil one ticket

Then she will select the winning ticket at random

The table shows information about the ticket used

<table>
<thead>
<tr>
<th>Colour of the ticket</th>
<th>Numbers used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 7</td>
<td>red 1 to 80</td>
</tr>
<tr>
<td>Year 8</td>
<td>blue 1 to 75</td>
</tr>
<tr>
<td>Year 9</td>
<td>yellow 1 to 90</td>
</tr>
<tr>
<td></td>
<td>245 Total</td>
</tr>
</tbody>
</table>

(a) What is the probability that the winning ticket will be blue?

Number of blue = 75

Total numbers = 245

Simplify $\frac{75}{245} = \frac{15}{49}$

or $0.306...$ or $31%$

1 mark

(b) What is the probability that the winning ticket will show number 39?

There will be red 39, blue 39, yellow 39

so 3 tickets with 39

$\frac{3}{245}$

1 mark

(c) The headteacher selects the winning ticket at random. She says: ‘The winning ticket number is 39.’ What is the probability that this winning ticket is blue?

Number is 39

$\rightarrow$ 3 tickets with 39.

blue is 1 of 3

$\frac{1}{3}$

1 mark
Mental Arithmetic Questions

1. Multiply 8.7 by 2
   \[8 \times 2 = 16\]
   \[0.7 \times 2 = 1.4\]
   \[17.4\]

2. A bat flies at an average speed of 32 kilometres an hour. At this speed, how far will it fly in 15 minutes?
   \[32\text{km} : 60\text{mins} \quad 16\text{km} : 30\text{mins} \quad 8\text{km} : 15\text{mins}\]

3. Multiply the brackets \((2x + 1)(x - 1)\)
   \[
   \begin{array}{c|cc|c}
   & 2x & 1 \\
   \hline
   x & 2x^2 & x \\
   -1 & -2x & -1 \\
   \end{array}
   \]
   \[2x^2 + x - 2x - 1\]

4. I'm thinking of a number. I call it \(t\). I half it and subtract five. Write an expression to show the result.
   \[
   \frac{t - 5}{7}
   \]

5. The first odd number is 1. What is the hundredth odd number?
   \[199\]
Owls

Owls eat small mammals. They regurgitate the bones and fur in balls called pellets.

<table>
<thead>
<tr>
<th>x</th>
<th>Number of mammals found in the pellet</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>f</td>
<td>Frequency</td>
<td>9</td>
<td>17</td>
<td>24</td>
<td>6</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>fx</td>
<td>No. of mammals X frequency</td>
<td>9</td>
<td>34</td>
<td>72</td>
<td>24</td>
<td>25</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) The table shows the contents of 62 pellets from long-eared owls.

(b) Calculate the mean number of mammals found in each pellet. Show your working and give your answer correct to 1 decimal place.

\[ \frac{\sum fx}{\Sigma f} = \frac{170}{62} = 2.74 \quad \text{...2.7...to 1 decimal place} \]

(c) There are about 10 000 long-eared owls in Britain. On average, a long eared owl regurgitates 1.4 pellets per day. Altogether, how many mammals do the 10 000 long-eared owls eat in one day? Show your working and give your answer to the nearest thousand.

Mammals in pellet \( x \) pellets per day \( x \) no. of owls
2.7 \( \times \) 1.4 \( \times \) 10 000 = 37 800

Rectangle Rest

The diagram shows a rectangle that just touches an equilateral triangle.

(a) Find the size of the angle marked \( x \)
Show your working.

1. Work out this one first
   \( 180 - 60 = 120^\circ \) angles on a straight line
2. Then this one
   \( 180 - 120 - 20 = 40^\circ \) angles in triangle
3. \( 180 - 90 - 40 \)
   \( 180 - 30 = 50^\circ \) on a straight line

(b) Now the rectangle just touches the equilateral triangle so that \( ABC \) is a straight line

Shaded triangle is BDE. For it to be Isosceles, \( \angle EDB = \angle EBD \)
So we need to show this

Remember to mark the size of angles that you have been given.

\( \angle EBD = 180 - 90 - 60 = 30^\circ \) angles on a straight line
\( \angle BDE = 180 - 60 = 120^\circ \) angles on a straight line
\( \angle EDB = 180 - 120 - 30 = 30^\circ \) angles in a triangle
Mental Arithmetic Questions

1. Add four to minus five.

\[-5 + 4 = -1\]

Think of number line or thermometer

2. What number should you add to minus three to get the answer five?

3. How many nought point fives are there in ten?

\[
0.5 \times 2 = 1 \\
1 \times 10 = 10
\]

4. On average, the driest place on earth gets only nought point five millimetres of rain every year. In total, how much rain would it expect to get in twenty years?

0

5. What is the sum of the angles in a rhombus?

All 4-sided shapes (quadrilaterals) have inside angles which sum to 360°
True or False

Pupils started to solve the equation $6x + 8 = 4x + 11$ in different ways.

For each statement below, tick (√) True or False.

You need to solve equation $(-8)$

(by $-4x$)

$6x + 8 = 4x + 11$
so $14x = 15x$

$6x + 8 = 4x + 11$
so $6x + 4x = 11 + 8$

A different pupil used trial and improvement to solve the equation

$6x + 8 = 4x + 11$

Explain why trial and improvement is not a good method to use

It would take a lot of trials
or Takes a long time
or You can solve it quickly by doing the same thing to both sides

Giraffe

A book gives this information:

A baby giraffe was born that was 1.58 metres high. It grew at a rate of 1.3 centimetres every hour.

Suppose the baby giraffe continued to grow at this rate.

About how many days old would it be when it was 6 metres high?
Show your working.

Starts $1.58m$ needs to grow $4.42m$ to reach $6m$.

$4.42m = 442cm$ (÷ 1.3 to find hrs)

$442 ÷ 1.3 = 340$ hrs (÷ 24 to find days)

$340 = 14.16$ days

1 mark
Mental Arithmetic Questions

1. It takes someone one and a half minutes to swim the length of the pool. How many lengths can I swim in 15 minutes?

\[
\begin{align*}
10 \times 1.5 &= 15 \\
\text{so } 15 \div 1.5 &= 10
\end{align*}
\]

2. Multiply minus eight by minus three.

\[
-8 \times -3 = 24
\]

3. If \(4x + 3 = 23\), what is the value of \(x\)?

\[
\begin{align*}
23 - 3 &= 4x \\
20 &= 4x \\
20 \div 4 &= x \\
x &= 6
\end{align*}
\]

4. I have a fair eight sided dice numbered 12 to 19. What is the probability that I will throw a prime number?

\[
P(\text{prime number}) = \frac{3}{8}
\]

Remember a prime number has only 2 factors, itself and 1. Possible primes are 13, 17 and 19.

5. What must I multiply \(n\) squared by to get \(n\) cubed?

\[
\begin{align*}
n^2 &= n \times n \\
n^3 &= n \times n \times n \\
\text{so } n^2 \times n &= n^3
\end{align*}
\]
**Rodents**

The scatter graph shows the average body length and average foot length of different species of rodents.

(a) What does the scatter graph tell you about the type of correlation between the body length and foot length for these rodents?

Positive Correlation

1 mark

(b) Draw a line of best fit on the scatter graph. Remember line of best fit does not need to go through 0, 0

1 mark

(c) If body length increased by 50mm, by approximately how many millimetres would you expect foot length to increase? Ring the correct value

2 15 50

1 mark

(d) An animal has a body length of 228mm, and a foot length of 22m. Is this animal likely to be one of these species of rodents?

Tick (√) Yes or No

Yes  √  No

1 mark

Explain your answer.

Use graph: from graph animal 228mm has foot length of 35 – 40mm.
So this is much bigger than 22mm.
∴ answer is No!

1 mark

**Centenarians**

People who live to be 100 years old are called centenarians.

In 1998 there were 135 000 centenarians
The ratio of male to female was 1:4

How many female centenarians were there in 1998?

Show your working

Male : female

1 : 4

So total number of parts = 1 + 4 = 5

1 part = \(\frac{135 000}{5} = 27 000\)

But females make up 4 parts, so \(4 \times 27 000 = 108 000\)

2 marks